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A MODEL FOR THE OBSERVED INTERMITTENCY OF A MEANDERING PLUME

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Summary

A proper description or prediction of the concentration field due to a dispersing atmospheric contaminant includes a measure of its variation. The ability of the intermittency of the concentration to provide this measure is discussed briefly. A model is presented which gives the profile of intermittency across a meandering plume in terms of the ratio of the instantaneous plume width and the limits of meander, and the intermittency existing within the instantaneous plume. The results show clearly that the profile can take many shapes, few of which may be represented by simple functions. The model results are compared against an available data set.

Introduction

For many purposes, the inadequacy of the mean concentration at a point as the sole descriptor or predictor for the concentration field due to a gaseous or particulate contaminant dispersing in the atmosphere is becoming increasingly recognised (Lockwood and Naguib, 1975; Chatwin, 1982). Other characteristics of the field are important, particularly fluctuations in the observed concentration at a point, which are the subject of this note. Thus, for instance, a potential toxic hazard could be specified by the probability with which a given concentration is exceeded for a range of particular averaging times. The process by which the atmosphere generates concentration fluctuations or intermittency in a plume is complex and impracticable to treat exactly, so there have been several attempts to describe plume characteristics, particularly the probability density function (PDF), by way of easily calculated two-parameter functions. The typical plume developed in the atmospheric boundary layer from a small source (such as a ruptured pipe) near the ground appears in plan as a gradually broadening ribbon which meanders with a superficially sinusoidal motion. A point down wind of the source may remain in the ribbon of material or, if the meander is sufficiently pronounced, the plume may pass back and forth laterally, submitting the point to an intermittent concentration. In addition to this intermittency of meander, there may be observed, with instruments of sufficient resolution, a within-plume intermittency with a generally distinctly smaller time scale. Without question, the dispersing material is subject to the whole range of ambient turbulent velocity scales, but the fact that the resulting intermittency can be so partitioned – to a first approximation – is clearly supported by the development of the plume as a readily observable entity. Wilson, Robins and Fackrell (1985) have used this technique with some success for prediction purposes as has Hanna (1986). A simple binary division is also supported by the nature of the mechanisms which control the plume growth: meandering is caused by turbulent eddies with dimensions greater than the plume width whilst the internal plume structure is determined by eddies smaller than this width. This point was strongly defended by Gifford (1974) in debate relating to his (1958) meandering plume model.

PDFs

Several two-parameter PDFs have been proposed as adequate tools for prediction purposes. Examples are a modified negative exponential (Hanna, 1984); a log-normal (Larsen, 1970); and a truncated Gaussian PDF, otherwise known as a clipped normal (Lewellen and Sykes, 1986). All these PDFs utilise the mean concentration as a parameter. In addition, the negative exponential employs the intermittency (defined formally here as the probability that zero concentration is observed), and the other two use the standard deviation of the concentration. Lewellen and Sykes (1986) have shown that the intermittency and the standard deviation are functionally related in the truncated Gaussian PDF. One sensible test for the suitability of this PDF is how well the theoretical relationship is matched by observation, a test employed by Ride (1987) which showed excellent agreement on the axis of a plume generated by a small point source near the ground in the open air. Similar relationships must exist for other two-parameter PDFs, since there cannot be more than two independent variables.

This brief discussion is intended to establish that the intermittency of the concentration is a useful measure of the concentration field and can be employed as a defining parameter in place of the standard deviation in PDFs for which the functional relationship between the two has been established and where it is not independently employed. The reason for using the intermittency in this way stems from the relative ease with which it can be computed from observations compared with the standard deviation, or from its emergence as an easily derivable statistic from some models (e.g., Ride, 1983). This is the justification for the model developed here which yields the cross-plume profile of intermittency given the size of the plume, relative to the width, and the within plume intermittency. The aim is to demonstrate that a simple model is capable of reproducing some of the important, observed characteristics of a plume. For practical predictions, it is currently necessary to turn to models which incorporate PDFs which are simply functional fits to observed fluctuations (e.g., Wilson, 1986).

Intermittency definitions and simple model assumptions

The intermittency of meander and the within-plume intermittency require separate definitions, since the first is a temporal phenomenon derived from the dynamics of the plume motion, whilst the second is treated here as a spatial property of the plume although it, too, will result in a time-varying concentration at a point.

Definition 1. The intermittency of meander, γ_M, at a point P is the probability that P lies outside the plume.

Definition 2. The within-plume intermittency, γ_P, is the probability that, at an arbitrary point within the plume, the concentration is observed as zero.

It is necessary to assume that γ_P is spatially distributed in an even manner across the plume so that a stationary observer at P would notice no change in its value regardless of the precise manner in which the plume accelerated in its meander.

The dynamic meandering of a plume at a fixed distance from the source can, like all oscilliatory motion, be described by the sum of a number of sinusoidal movements, one of which is usually dominant. The easiest way to describe a sinusoidal oscillation is by Simple Harmonic Motion (SHM). Concentrate attention on the dominant frequency and consider a lateral horizontal cross section of a symmetrical plume of instantaneous width $2p$ which meanders with SHM under the influence of large eddies symmetrically between limits which are a distance of $2m$ apart, with $m > p$.

The SHM has amplitude $m-p$. Figure 1 illustrates the scenario. One of three situations can exist at the point P, which is located a distance x from the axis of meander. First, if $|x| < 2p-m$, the point will remain in the plume at all times. Secondly, if $|x| < m-2p$ and $2p < m$, the point will experience the passage of both plume edges as it passes in each direction. Lastly, if $|x| > |2p-m|$, P will experience the passage of only one edge, being alternately engulfed from one direction and cleared from the other. This last case is illustrated by the two possible configurations which give rise to it in Fig. 1.

The first situation is trivial. Consider the second. Starting from rest, the velocity dy/dt of the edge nearest the axis is given by the SHM result (e.g., Rutherford, 1951, p. 48), with a translation of the axis,

$$dy/dt = -(2\pi/T)\left[(m-p)^2-(y+p)^2\right]^{1/2} \tag{1}$$

where T is the period of meander. The time $T_N(x)$ for the edge to reach x from the extreme position is obtained by integrating eqn. (1) from $y=m-2p$ to $y=x$ so:

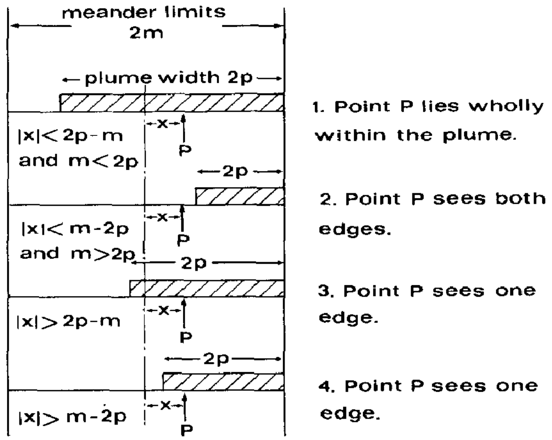

Fig. 1. A pictorial representation of the meandering plume model. The instantaneous plume is shown as a shaded block at an extreme limit of its travel. Cases 3 and 4 reduce to $|x| > |2p-m|$.

$$T_N(x) = (T/(2\pi))\{\pi/2 - \arcsin[(x+p)/|m-p|]\} \tag{2}$$

Similarly, the time $T_F(x)$ for the edge farthest from the axis to reach P is given by:

$$T_F(x) = (T/(2\pi))\{\pi/2 - \text{arscin}[(x-p)/|m-p|]\} \tag{3}$$

The period that P resides in the plume for half a swing is simply $T_F(x) - T_N(x)$. Point P resides in the plume during the reverse half swing for a period $T_F(-x) - T_N(-x)$. Adding these two periods and dividing by T yields $1-\gamma_M$. In a similar manner, for the third case where only one edge traverses the point, γ_M is calculated by putting $T_N(x)=0$ and $T_F(-x)=\frac{1}{2}T$, i.e., the point or its mirror position starts the half cycle within the plume. It is clear that the intermittency of meander depends, in this model, only on the ratio p/m ($=r$, say).

The total observed intermittency, γ, is given by:

$$\gamma = \gamma_M + (1-\gamma_M)\cdot\gamma_P \tag{4}$$

Results and discussion

A computer program was writeen by the author to calculate γ for various input values of r and p. The results are shown in Fig. 2. A notable characteristic is that the total intermittency achieves a maximum on the axis for plumes with a relatively large meander ($r<0.5$); they move fastest there and so spend more time near their outer limits. This situation is reversed for plumes which meander less ($r>0.5$). These features are deterministic in nature, and where they

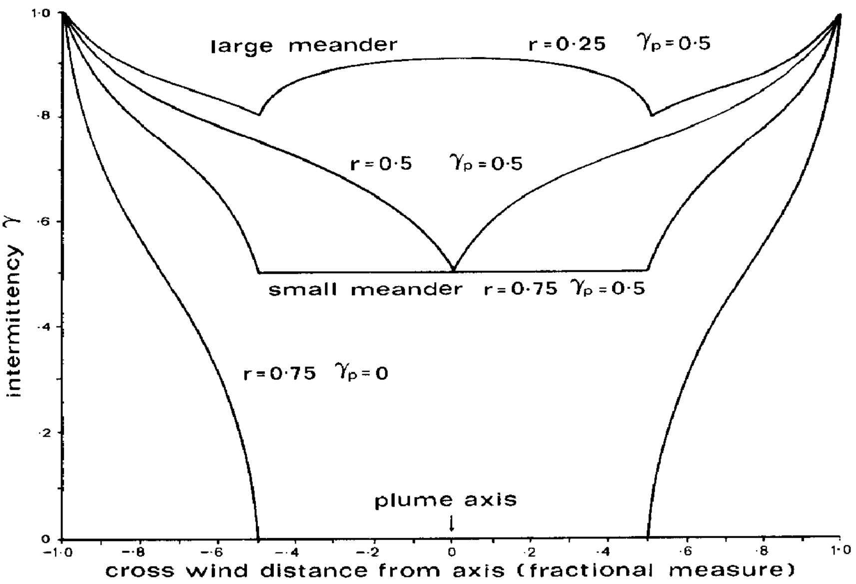

Fig. 2. Examples of model plume intermittency profiles. r is the ratio of the instantaneous plume to the width of meander. γ_P is the within-plume intermittency.

are observed in real plumes they should not be dismissed as random variations. The sharp discontinuities in the slopes of the computed curves are a consequence of employing a SHM model, and perhaps would be evident in real plumes only when the sampling period does not exceed the meander period; variations in the characteristics of meander would smooth the discontinuities for longer sampling periods. They would also be smoothed when observed by sensors with a finite averaging time. It is clear that the intermittency profiles cannot be adequately modelled in general by simple distributions like the Gaussian PDF.

Suitable observations with which to compare the model are rare. Wilson, Robins and Fackrell (1985) show a cross plume profile of intermittency measured in a wind tunnel, but the meander in a wind tunnel is severely restricted by the absence of large eddies; also, their axial measurements, although highly resolved (300 Hz), display very little within-plume intermittency. Hanna (1984) gives intermittency data for a section across a meandering smoke plume generated by a continuous source of oil fog. Values of $r = 0.65$ and $\gamma_P = 0.45$ were used in the model to produce a comparison with his data. This comparison is shown in Fig. 3 and is judged to be good.

If the instantaneous plume possesses an axial maximum of concentration (as is usually assumed for modelling purposes), the profile of concentration across the time averaged plume should exhibit – according to the model – two maxima which are separated by a centrally located minimum where the plume is travelling fastest. Although they are not equal in size, two concentration

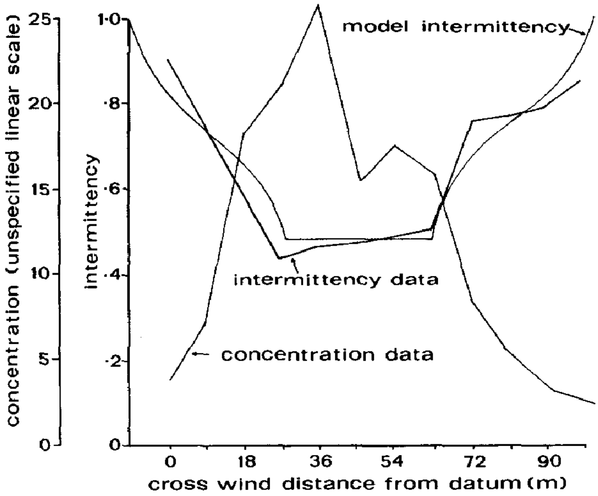

Fig. 3. Observed intermittency and concentration in a meandering plume (after Hanna, 1984) and intermittency from the simple model.

maxima are observed in Hanna's data in Fig. 3, symmetrically disposed about the model's axis of meander. This correspondence is taken as additional support for the model's ability to reproduce essential characteristics of the concentration profile and justifies the assumption that intermittency can be partitioned into two components, representative of the different ends of a spectrum which in reality is continuous. The high degree to which this approximation is seemingly valid is perhaps surprising, but if a simple two parameter PDF can describe the concentration distribution well then an equivalent valid approximation of the physical processes which lead to it should not be unexpected.

Sawford and Stapountzis (1986) developed an analytical PDF model for concentration within a meandering plume based on assumptions of Gaussian distributions for the position of the instantaneous plume and the cross wind distribution of concentration within it. They attribute the generally poor fit of the model to wind tunnel measurements to inadequacies in predicting (in present parlance) r, and the neglect of small scale structure, but the problems of predicting it (and r) for practical use remain. However, Sawford and Stapountzis conjecture that the profile of small scale structure across an instantaneous plume should remain sensibly self similar. This is a reasonable conjecture and is implicit in the present model.

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